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Paper Code : BSM202 Mathematics - IIB

UPID : 002006

Time Allotted : 3 Hours *Full Marks :70*

The Figures in the margin indicate full marks.

Candidate are required to give their answers in their own words as far as practicable

Group-A (Very Short Answer Type Question)

1. Answer any ten of the following : [1 x 10 = 10]

(I) Find the value of $\lim_{z \to i} \frac{iz+1}{z-i}$

(II) Find the residue of $f(z) = e^{-\frac{1}{z}}$ at $z = 0$.

(III) Find the value of the integral $\oint_c (xdy - ydx)$ where C is the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(IV) Find the IF of the differential equation $\frac{dy}{dx} - 3y = \sin 2x$.

(V) Write the general solution of the ordinary differential equation $\frac{d^2y}{dx^2} + 4y = 0$.

(VI) Is the function $f(z) = |z|^2$ continuous everywhere?

(VII) Find $\frac{1}{D^2+4}(x)$

(VIII) If $f(z) = u + iv$ is an analytic function in a finite region and $u = x^3 - 3xy^2$, then find v.

(IX) Find the residue of $\frac{z^2}{z^2+a^2}$ at $z = ia$.

(X) Find the value of $\int_0^{\frac{\pi}{2}} \int_0^{\sin\theta} r^2 \sin\theta \, dr d\theta$

(XI) Find the value of $\iint_S \vec{F}.\hat{n}\, dS$, where $F = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$, where S is the surface of cube given by $x = 0, x = 1; y = 0, y = 1; z = 0, z = 1$.

(XII) Find the singular solution of $y = px - \frac{1}{4}p^2$.

Group-B (Short Answer Type Question)

Answer any three of the following : [5 x 3 = 15]

2. [5]

Prove that $\lim_{Z \to 0} \frac{\bar{Z}}{Z}$ does not exist.

3. Evaluate $\oint_{|z|=1} \frac{e^{3z}}{(4z-\pi i)^3} dz$. [5]

4. Solve: [5]

$$(xy \sin xy + \cos xy)ydx + (xy \sin xy - \cos xy)xdy = 0$$

5. Show that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$ [5]

6. Solve: $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + 4y = x\, Sin(\log x)$ [5]

Group-C (Long Answer Type Question)

Answer any three of the following : [15 x 3 = 45]

7\. (a) Show that $(3x + 4y + 5)dx + (4x - 3y + 3)dy = 0$ is an exact equation and hence solve it. [5]

(b) Solve: $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$. [5]

(c) Solve: $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$ [5]

8\. (a) Prove that $J_0' = -J_1$. [4]

(b) Express $J_4(x)$ in terms of J_0 and J_1 [5]

(c) Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2} + a^2y = Sec\, ax, (a \neq 0)$ [6]

9\. (a) Use the transformation $u = x + y$ and $uv = y$, evaluate the double integration $\int_0^1 dx \int_0^{1-x} e^{\frac{y}{x+y}} dy$. [5]

(b) Evaluate $\iiint (x + y + z + 1)^4 dxdydz$, over the region bounded $x \geq 0$, $y \geq 0, z \geq 0, x + y + z \leq 1$. [5]

(c) Evaluate $\iiint z^2 dxdydz$, extended over the hemisphere $z \geq 0, x^2 + y^2 + z^2 \leq a^2$. [5]

10\. (a) Determine the analytic function $f(z) = u + iv$ whose imaginary part is $v(x, y) = e^x \sin y$. [5]

(b) Prove that $u(x, y) = \frac{1}{2}\log(x^2 + y^2)$ is harmonic and find its conjugate harmonic function $v(x, y)$ such that $f(z) = u + iv$ is analytic. [5]

(c) Show that the transformation$f(z) = \frac{z+i}{z-i}$ maps the interior of the circle $|w| = 1\; i.e. |w| \leq 1$ into the lower half plane $I(z) \leq 0$. [5]

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11. (a) Prove that $(x+y+1)^{-4}$ is an integrating factor of the differential equation [5]

$$(2xy - y^2 - y)dx + (2xy - x^2 - x)dy = 0$$

and hence solve it.

(b) Solve: $3ydx - 2xdy + x^2y^{-1}(10ydx - 6xdy) = 0$ [5]

(c) Solve: $\frac{dy}{dx} + y = y^3(cosx - sinx)$. [5]

*** END OF PAPER ***